School Bus Routing with Collaboration

Hao Hao

In collaboration with Zihan Li, Neha, Hai Wang, Peter Zhang
Motivation

• For Allegheny County school districts, school bus transportation is a major expense

• The current routing policies are often designed sub-optimally, resulting in long bus rides and high costs

• In this multi-school system, routing decisions are made individually, without leveraging the benefit of route sharing among schools

• Formally, the school bus routing problem (SBRP) with collaboration aims to generate optimal routes and schedules for each bus while allowing collaboration across multiple schools, so that the number of buses and the total travel time are minimized
Challenges

• First, SBRP for each school without sharing is already a class of hard combinatorial optimization problems. In which three decisions must be made:
  1) Selecting the subset of stops to visit amount all potential stops
  2) Assigning each student to a stop
  3) Generate optimal routing and scheduling for each bus under time and capacity constraints while minimizing travel time

State-of-the-art methods typically solve this problem using metaheuristics, including local search.

• To further solve collaborative routing problem between multiple schools, an additional decision is required to select the subsets of schools that yield the highest benefits from collaboration. In practice, it is infeasible to consider the combinatorically many subsets of schools to find the optimal combinations, as each subset requires solving its own instance of SBRP.

• The above challenges inspired a more computational feasible approach for screening the set of schools for collaboration

• Specifically, we wish to learn a model-based estimation of the optimal routing time for SBRP instances. The learned model will allow SBRP estimates without the high computational cost of solving the combinatorial optimization problems, therefore enabling quick identification of the subset of schools that can benefit from sharing buses and routes.
New Approach: Online learning of Optimal TSP Route Length

This approach is inspired by the Beardwood–Halton–Hammersley (BHH) model:

Let $X_1, X_2, \ldots, X_n$ be independent and uniform random variables in the square $[0, \sqrt{a}]^2$ and let $L(X_1, X_2, \ldots, X_n)$ be the length of the shortest traveling salesman problem (TSP) path through these points. There exists estimates of optimal TSP for some constant $\beta$ and $c$.

$$L(X_1, X_2, \ldots, X_n) \leq \beta \sqrt{n}a + c \quad (1)$$

We extend the BHH model to a general model for estimating the optimal TSP route length in real-world road networks. Similar to the sample graph shown in Fig 2, consider a graph representation of any transportation network of $|V| = n$ nodes, with the coordinate of node $i$ as $V_i$. We define the following vector

$$X(n, G) = [\sqrt{n} \lambda_1(G), \sqrt{n} \lambda_2(G), 1]^T \quad (2)$$

Where $G(i, j) = <V_i, V_j>$ describes the inner product space of coordinate vector of all nodes, and $\lambda_1(G), \lambda_2(G)$ are the first and second largest eigenvalues of $G$. Given $X(n, G)$ of a road network, we wish to learn an estimate of its optimal TSP route length as

$$\hat{y} = W^TX(n, G) \quad (3)$$

Where the true TSP route length is solved as a combinatorial optimization problems by Tabu search using the distance matrix $D$ as

$$y = TSP(n, D) \quad (4)$$
New Approach: Online learning of Optimal TSP Route Length

- Given instances of the TSP: \( \{y, X\} \), we wish to learn the following general model for TSP estimate:
  \[
  \hat{y} = W^T X
  \]

However, as previously mentioned, computing an instance of the TSP solution, \( y \) is costly. Therefore it is prohibitively slow to gather enough data to learn the model parameters at once. As a result, we propose to learn in an online manner, for which the TSP instances come sequentially. This allows the model to learn with a small batch of data at a time.

- The stochastic gradient descent with mini-batch update can be performed for online learning:

```
Algorithm 1 Gradient Descent with Mini-batch Update

Data: \((X_1, y_1), (X_2, y_2), ..., (X_T, y_T)\)
Initialization: model parameter \( W = (W_1, W_2, ..., W_K); j = 0; \) batch Size, \( b; \) learning rate, \( \eta_0; \)
for Every batch, \( t=1:b:T \) do
    \( j = j+1 \)
    for Every data in batch: \( t'=t:t+t+b-1 \) do
        \( \Delta W_k = 0 \)
        for Every Parameter: \( k = 1:K \) do
            Compute \( \nabla_{W_k} \text{Loss}(f(X_t, W), y_t) \)
            \( \Delta W_k = \Delta W_k + \frac{1}{b} \nabla_{W_k} \text{Loss}(f(X_t, W), y_t) \)
        end
        for Every Parameter: \( k = 1:K \) do
            \( W_k = W_k - \eta \Delta W_k \)
        end
    end
end
```
Application Scenarios

• The proposed model-based TSP estimation by online learning can be applied to any road network. As a result, the potential end-product is a general tool that takes input of a graph representation of a road network of any region in the world and produce the optimal TSP route in an efficiency manner.

• In addition, for the purpose of its original application in SBRP with collaboration, the TSP estimation model can be extended to vehicle routing problem (VRP). As a result, given the low-complexity of estimation by learnt model, it is now feasible to consider the combinatorially many subsets of schools to find the optimal combinations that maximize the benefit.